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Brane-world cosmology in higher derivative gravity or warped compactification in the next-to-leading order of AdS/CFT correspondence

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Abstract: The general model of higher-derivative (HD) gravity is considered. The search of brane-world cosmology in such theory is presented when bulk is d5 AdS and boundary is spherical, hyperbolic or flat (single) brane. It is found the wide range of theory parameters where such cosmology may be realized. Special attention is paid to the version of HD theory representing SG dual of $\mathcal{N}=2$ Sp(N) SCFT (in next-to-leading order of large-N expansion). In particular, it is shown that inflationary brane Universe does not occur for SG dual while hyperbolic brane occurs (which was not possible in leading order). The quantum effects of CFT living on the brane (via the corresponding conformal anomaly induced effective action) may qualitatively change the results of classical analysis. There appears inflationary (or hyperbolic) brane Universe induced by only quantum effects. In AdS/CFT correspondence (next-to-leading order) the addition of such CFT effective action (in some energy region) is naturally explained in terms of holographic renormalization group. It results in the possibility of quantum creation of inflationary brane Universe (with small rate) even for SG dual.

Keywords: Superstrings and Heterotic Strings, D-branes, AdS-CFT Correspondance.

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Contents

1. Introduction

In brane-world scenarios one lives on the boundary (observable Universe) where gravity is trapped [1]. Such brane is embedded in higher-dimensional bulk space. The investigation of cosmological aspects of brane-worlds [2, 3] (see also references therein) shows that at some circumstances the inflationary Universe could be realized on the brane. Even the experimental tests to search for higher-dimensional deviations of our world due to bulk/boundary structure may be proposed [4]. However, the study of such brane cosmology has been done so far almost exclusively for Einstein or dilatonic gravities. On the same time, higher-derivative (HD) gravity represents very natural generalization of general relativity. It enjoys various nice features like renormalizability [5] and asymptotic freedom in four dimensions (see [7] for introduction and review), possibility of self-consistent compactification on quantum and classical levels (see examples in [8]), sufficiently small corrections to Newton potential at reasonable range of parameters, etc. Moreover, HD terms are typical for string effective action in the derivatives expansion [9, 10]. Hence, HD gravity in higher dimensions represents the interesting model where brane-world cosmology should be investigated and the trapping of gravity should be discussed. Note that propagator in such theory is qualitatively different from the one in general relativity that is why some new phenomena may be expected.

The important remark is in order. In the widely accepted versions of brane-world scenario [1, 2, 3] one studies $d+1$-dimensional gravity coupled to a brane in the formalism where normally two free parameters (bulk cosmological constant and brane tension) present. Adding of d-dimensional Gibbons-Hawking boundary action and brane cosmological constant term to action one can get de Sitter brane in AdS bulk even in Einstein gravity. The position of such brane is fixed in terms of brane tension.

Our ideology is somehow different, in the spirit of refs. [12, 22]. Namely, one considers the addition of surface counterterms which make variational procedure to be well defined and eliminate the leading divergences of action. Brane cosmological constant (or brane tension) is not considered as free parameter (as it was in original brane-world scenario) but it is fixed by condition of finiteness of spacetime when brane goes to infinity. In such approach, the possibility of cosmological de Sitter brane-world in Einstein theory is eliminated. However, as we explain below in theories different from Einstein gravity this possibility is not ruled out thanks to other free parameters of theory.

Our purpose in the present work will be to study the brane-world cosmology in d5 HD gravity without (or with) quantum corrections. We consider general model with arbitrary coefficients (section 2) and derive bulk and boundary equations of motion. The explicit structure of surface counterterms is very important in such derivation.

The solution of bulk equation of motion gives d5 AdS. Then, the brane equation of motion is discussed. It gives the restrictions to HD gravity parameters from the condition of realization of spherical, hyperbolic or flat branes on the boundary. The corresponding radius is derived when it exists. The specific versions of HD gravity like Weyl or Gauss-Bonnet theories appear in this formalizm as particular examples.

It is very important that there may be twofold point of view to HD gravity. From one side, this is just alternative to general relativity. From another side, within AdS/CFT correspondence [6] some versions of HD gravity represent SG duals where Einstein and cosmological terms are of leading order and HD terms are of next-to-leading order in large-N approximation. The explicit example of that sort is presented in section 2 (SG dual to $\mathcal{N} = 2$ SCFT). Brane-world cosmology for such theory naturally appears as warped compactification in the next-to-leading order of AdS/CFT correspondence. It is shown that in such situation the creation of spherical brane is impossible in leading as well as in next-to-leading order of large-N expansion. On the same time, the account of next-to-leading order terms makes possible the creation of hyperbolic brane living in d5 AdS bulk (this effect was prohibited in the leading order of AdS/CFT correspondence).

In section 3 we investigate the modification of above scenario when quantum CFT is living on the brane. This is done via the anomaly induced effective action. (The corresponding study for bulk Einstein gravity has been done in refs. [11, 12, 22].[2] In fact, this puts world-brane scenario in form of warped compactification in AdS/CFT set-up as the corresponding RG flow.) We show that for range of HD gravity parameters where classical consideration does not give inflationary or

[2] In usual 4d world the anomaly driven inflation has been proposed in refs. [13].

hyperbolic branes, the quantum brane matter effects improve the situation: purely quantum creation of inflationary or hyperbolic branes in d5 AdS occurs (like in Einstein gravity). On the same time if the phenomenon already existed on classical level then quantum corrections do not destroy it. The bulk clearly is not modified.

For SG dual of $\mathcal{N}=2$ SCFT theory such picture may be naturally understood in terms of holographic RG description [23] in AdS/CFT set-up. That is two dual descriptions (SG dual and QFT dual) are matched together in some energy region into the global representation of some RG flow. It is interesting that both sides are given in next-to-leading order of large-N expansion as conformal anomaly coefficients for SCFT under discussion include not only quadratic but also linear terms on N. Then, it is shown that in such theory the spherical brane in d5 AdS is created (the role of conformal anomaly coefficients is dominant). Without quantum CFT living on the brane it was impossible. For hyperbolic brane creation the qualitative results are not changed if compare with previous section. Finally, in the last section we present brief summary of results and mention some possible developments along the direction under consideration.

2. Brane-world cosmology in HD gravity

We consider 5d spacetime whose boundary is 4-dimensional sphere S_4, which can be identified with a D3-brane, four-dimensional hyperboloid H_4, or four-dimensional flat space R_4. The bulk part is given by 5-dimensional euclidean anti-de Sitter space AdS_5

$$ds^2_A dS_5 = dy^2 + \sinh^2 \frac{y}{l} d\Omega_4^2 \,. \tag{2.1}$$

Here $d\Omega_4^2$ is given by the metric of S_4, H_4 or R_4 with unit radius. One also assumes the boundary (brane) lies at $y = y_0$ and the bulk space is given by gluing two regions given by $0 \leq y < y_0$.

One starts with the following action:

$$S = \int d^5x \sqrt{\hat{G}} \left\{ a\hat{R}^2 + b\hat{R}_{\mu\nu}\hat{R}^{\mu\nu} + c\hat{R}_{\mu\nu\xi\sigma}\hat{R}^{\mu\nu\xi\sigma} + \frac{1}{\kappa^2}\hat{R} - \Lambda \right\} . \tag{2.2}$$

Here the conventions of curvatures are given by

$$\begin{aligned}
R &= g^{\mu\nu} R_{\mu\nu} \,, \\
R_{\mu\nu} &= -\Gamma^\lambda_{\mu\lambda,\kappa} + \Gamma^\lambda_{\mu\kappa,\lambda} - \Gamma^\eta_{\mu\lambda}\Gamma^\lambda_{\kappa\eta} + \Gamma^\eta_{\mu\kappa}\Gamma^\lambda_{\lambda\eta} \,, \\
\Gamma^\eta_{\mu\lambda} &= \frac{1}{2} g^{\eta\nu} \left(g_{\mu\nu,\lambda} + g_{\lambda\nu,\mu} - g_{\mu\lambda,\nu} \right) .
\end{aligned} \tag{2.3}$$

When $a = b = c = 0$, the action (2.2) becomes that of the Einstein gravity:

$$S = \int_{M_{d+1}} d^{d+1}x \sqrt{\hat{G}} \left\{ \frac{1}{\kappa^2}\hat{R} - \Lambda \right\} . \tag{2.4}$$

If we choose

$$a = \frac{1}{6}\hat{c}\,, \qquad b = -\frac{4}{3}\hat{c}\,, \qquad c = \hat{c} \tag{2.5}$$

the HD part of action is given by the square of the Weyl tensor $C_{\mu\nu\rho\sigma}$:

$$S = \int d^5x\sqrt{\hat{G}}\left\{\hat{c}\hat{C}_{\mu\nu\xi\sigma}\hat{C}^{\mu\nu\xi\sigma} + \frac{1}{\kappa^2}\hat{R} - \Lambda\right\}. \tag{2.6}$$

It is interesting that the string theory dual to $\mathcal{N} = 2$ superconformal field theory is presumably IIB string on $AdS_5 \times X_5$ [16] where $X_5 = S^5/\mathbb{Z}_2$. (The $\mathcal{N} = 2$ Sp(N) theory arises as the low-energy theory on the world volume on N D3-branes sitting inside 8 D7-branes at an O7-brane). Then in the absence of Weyl term, $1/\kappa^2$ and Λ are given by

$$\frac{1}{\kappa^2} = \frac{N^2}{4\pi^2}\,, \qquad \Lambda = -\frac{12N^2}{4\pi^2}\,. \tag{2.7}$$

This defines the bulk gravitational theory dual to super YM theory with two supersymmetries. The Riemann curvature squared term in the above bulk action may be deduced from heterotic string via heterotic-type I duality [17], which gives $\mathcal{O}(N)$ correction:

$$a = b = 0\,, \qquad c = \frac{6N}{24 \cdot 16\pi^2}\,. \tag{2.8}$$

Hence, HD gravity with above coefficients defines SG dual of super Yang-Mills theory (with two supersymmetries) in next-to-leading order of AdS/CFT correspondence [6].

Using field redefinition ambiguity [18] one can suppose that there exists the scheme where $R^2_{\mu\nu\alpha\beta}$ may be modified to $C^2_{\mu\nu\alpha\beta}$ in the same way as in ref. [19]. Then, the action (2.4) is presumably the bulk action (in another scheme) dual to $\mathcal{N} = 2$ SCFT.

Let us start from the bulk equations of motion. First we investigate if the equations of motion for the general action (2.2) have a solution which describes anti-de Sitter space, whose metric is given by

$$ds^2 = \hat{G}^{(0)}_{\mu\nu}dx^\mu dx^\nu = \frac{l^2}{4}\rho^{-2}d\rho d\rho + \sum_{i=1}^{4}\rho^{-1}\eta_{ij}dx^i dx^j\,. \tag{2.9}$$

When we assume the metric in the form (2.9), the scalar, Ricci and Riemann curvatures are given by

$$\hat{R}^{(0)} = -\frac{20}{l^2}\,, \qquad \hat{R}^{(0)}_{\mu\nu} = -\frac{4}{l^2}G^{(0)}_{\mu\nu}\,, \qquad \hat{R}^{(0)}_{\mu\nu\rho\sigma} = -\frac{1}{l^2}\left(G^{(0)}_{\mu\rho}G^{(0)}_{\nu\sigma} - G^{(0)}_{\mu\sigma}G^{(0)}_{\nu\rho}\right)\,, \tag{2.10}$$

which tell that these curvatures are covariantly constant. Then in the equations of motion from the action (2.2), the terms containing the covariant derivatives of the curvatures vanish and the equations have the following form:

$$\begin{aligned} 0 = &-\frac{1}{2}G^{(0)}_{\zeta\xi}\left\{a\hat{R}^{(0)2} + b\hat{R}^{(0)}_{\mu\nu}\hat{R}^{(0)\mu\nu} + c\hat{R}^{(0)}_{\mu\nu\rho\sigma}\hat{R}^{(0)\mu\nu\rho\sigma} + \frac{1}{\kappa^2}\hat{R}^{(0)} - \Lambda\right\} + \\ &+ 2aR^{(0)}R^{(0)}_{\zeta\xi} + 2b\hat{R}^{(0)}_{\mu\zeta}\hat{R}^{(0)\mu}_{\xi} + 2c\hat{R}^{(0)}_{\zeta\mu\nu\rho}\hat{R}^{(0)\mu\nu\rho}_{\xi} + \frac{1}{\kappa^2}\hat{R}^{(0)}_{\zeta\xi}\,. \end{aligned} \tag{2.11}$$

Then substituting eqs. (2.10) into (2.11), one gets

$$0 = \frac{80a}{l^4} + \frac{16b}{l^4} + \frac{8c}{l^4} - \frac{12}{\kappa^2 l^2} - \Lambda \,. \tag{2.12}$$

Equation (2.12) can be solved with respect to l^2 if

$$\frac{144}{\kappa^4} - 16\left\{20a + 4b + 2c\right\}\Lambda \geq 0 \tag{2.13}$$

which can been found from the determinant in (2.12). Then we obtain [20]

$$l^2 = -\frac{12/\kappa^2 \pm \sqrt{\frac{144}{\kappa^4} - 16\left\{20a + 4b + 2c\right\}\Lambda}}{2\Lambda} \,. \tag{2.14}$$

The sign in front of the root in the above equation may be chosen to be positive which corresponds to the Einstein gravity ($a = b = c = 0$). For SG dual of $\mathcal{N} = 2$ Sp(N) theory, we find from (2.8)

$$\frac{1}{l^2} = 1 + \frac{1}{24N} + \mathcal{O}\left(N^{-2}\right) . \tag{2.15}$$

Now, let us discuss the surface terms in HD gravity on the chosen background

$$ds^2 \equiv \hat{G}_{\mu\nu} dx^\mu dx^\nu = \frac{l^2}{4}\rho^{-2} d\rho d\rho + \sum_{i=1}^{d} \hat{g}_{ij} dx^i dx^j \,, \quad \hat{g}_{ij} = \rho^{-1} g_{ij} \,. \tag{2.16}$$

If the boundary of AdS_5 lies at $\rho = \rho_0$, the variation δS contains the derivative of $\delta\hat{g}^{ij}$ with respect to ρ, which makes the variational principle ill defined. In order that the variational principle is well defined on the boundary, the variation of the action should be written in the form of

$$\delta S = \int d^5x \sqrt{\hat{G}}\, \delta\hat{g}^{ij} \times (\text{eq. of motion}) + \int_{\rho=\rho_0} d^4x \sqrt{\hat{g}}\, \delta\hat{g}^{ij} \left\{\cdots\right\} \tag{2.17}$$

after using the partial integration. If we put $\{\cdots\} = 0$ for $\{\cdots\}$ in (2.17), we could obtain the boundary condition. If the variation of the action on the boundary contains $(\delta\hat{g}^{ij})'$, however, we cannot partially integrate it with respect to ρ on the boundary to rewrite the variation in the form of (2.17) since ρ is the coordinate expressing the direction perpendicular to the boundary. Therefore the "minimum" of the action is ambiguous. Such a problem was well studied by Gibbons and Hawking in [14] for the Einstein gravity ($a = b = c = 0$). The boundary term was added to the action, which cancels the variation:

$$S_b^{\rm GH} = -\frac{2}{\tilde{\kappa}^2} \int_{\rho=\rho_0} d^4x \sqrt{\hat{g}}\, D_\mu n^\mu \,. \tag{2.18}$$

Here n^μ is the unit vector normal to the boundary. In the coordinate choice (2.16), the action (2.18) has the form

$$S_b^{\rm GH} = -\frac{2}{\tilde{\kappa}^2}\int_{\rho=\rho_0} d^4x\sqrt{\hat{g}}\frac{\rho}{l}\hat{g}_{ij}\left(\hat{g}_{ij}\right)' . \tag{2.19}$$

Then the variation over the metric $\hat{g}_{ij}$ gives

$$\begin{aligned}\delta S_b^{\rm GH} &= -\frac{2}{\tilde{\kappa}}\int_{\rho=\rho_0} d^4x\sqrt{\hat{g}}\frac{\rho}{l}\times \\ &\times\left[\delta\hat{g}^{ij}\left\{-\hat{g}_{ik}\hat{g}_{il}\left(\hat{g}_{kl}\right)' - \frac{1}{2}\hat{g}_{ij}\hat{g}_{kl}\left(\hat{g}_{kl}\right)'\right\} + \hat{g}_{ij}\left(\delta\hat{g}_{ij}\right)'\right] .\end{aligned} \tag{2.20}$$

From the other side, the surface terms in the variation of the bulk Einstein action ($a = b = c = 0$ in (2.4)) have the form

$$\begin{aligned}\delta S^{\rm Einstein} &= \int d^5x\sqrt{\hat{G}}\delta\hat{g}^{ij}\times(\text{Einstein equation}) + \\ &+\frac{1}{\kappa^2}\int_{\rho=\rho_0} d^4x\sqrt{\hat{g}}\frac{2\rho}{l}\left[\hat{g}'_{ij}\delta\hat{g}^{ij} + \hat{g}_{ij}\left(\delta\hat{g}^{ij}\right)'\right] .\end{aligned} \tag{2.21}$$

Then we find the terms containing $\left(\delta\hat{g}^{ij}\right)'$ in (2.20) and (2.21) are cancelled with each other.

We also need the counterterms, besides Gibbons-Hawking term (2.18), to cancell the divergence coming from the infinite volume of AdS. Such a kind of counterterms can be given by the local quantities on the 4-dimensional boundary. In [20], the surface counterterms are discussed for higher derivative gravities in all detail. Note that they are relevant also for quantum cosmology [15].

We also add the surface terms $S_b^{(1)}$ corresponding to Gibbons-Hawking term (2.18) and $S_b^{(2)}$ which is the leading counterterm corresponding to the vacuum energy on the brane:

$$\begin{aligned}S_b &= S_b^{(1)} + S_b^{(2)} , \\ S_b^{(1)} &= \int d^4x\sqrt{\hat{g}}\left[4\tilde{a}\hat{R}D_\mu n^\mu + 2\tilde{b}\left(n_\mu n_\nu\hat{R}^{\mu\nu}D_\sigma n^\sigma + \hat{R}_{\mu\nu}D^\mu n^\nu\right) + \right. \\ &\qquad\left. + 8\tilde{c}n_\mu n_\nu\hat{R}^{\mu\tau\nu\sigma}D_\tau n_\sigma - \frac{2}{\tilde{\kappa}^2}D_\mu n^\mu\right] , \\ S_b^{(2)} &= -\eta\int d^4x\sqrt{\hat{g}} .\end{aligned} \tag{2.22}$$

In [20], in order to cancell the leading order divergence, which appears when the brane goes to infinity, we got

$$\eta = -\frac{32T}{l^3} + \frac{8}{l\kappa^2} + \frac{4\tilde{T}}{l^3} - \frac{2}{l\tilde{\kappa}^2} . \tag{2.23}$$

Here

$$T = 10a + 2b + c\,, \qquad \tilde{T} = 10\tilde{a} + 2\tilde{b} + \tilde{c}\,. \tag{2.24}$$

Note that unlike to standard brane-world scenarios η is not free parameter.

The metric of S_4 with the unit radius is given by

$$d\Omega_4^2 = d\chi^2 + \sin^2\chi d\Omega_3^2\,. \tag{2.25}$$

Here $d\Omega_3^2$ is described by the metric of 3-dimensional unit sphere. If we change the coordinate χ to σ by

$$\sin\chi = \pm\frac{1}{\cosh\sigma}\,, \tag{2.26}$$

one obtains

$$d\Omega_4^2 = \frac{1}{\cosh^2\sigma}\left(d\sigma^2 + d\Omega_3^2\right). \tag{2.27}$$

On the other hand, the metric of the 4-dimensional flat euclidean space is given by

$$ds_{4\mathrm{E}}^2 = d\zeta^2 + \zeta^2 d\Omega_3^2\,. \tag{2.28}$$

Then by changing the coordinate as

$$\zeta = \mathrm{e}^{\sigma}\,, \tag{2.29}$$

one gets

$$ds_{4\mathrm{E}}^2 = \mathrm{e}^{2\sigma}\left(d\sigma^2 + d\Omega_3^2\right). \tag{2.30}$$

For the 4-dimensional hyperboloid with the unit radius, the metric is given by

$$ds_{\mathrm{H4}}^2 = d\chi^2 + \sinh^2\chi d\Omega_3^2\,. \tag{2.31}$$

Changing the coordinate χ to σ

$$\sinh\chi = \frac{1}{\sinh\sigma}\,, \tag{2.32}$$

one finds

$$ds_{\mathrm{H4}}^2 = \frac{1}{\sinh^2\sigma}\left(d\sigma^2 + d\Omega_3^2\right). \tag{2.33}$$

Motivated by (2.27), (2.30) and (2.33), one takes the metric of 5-dimensional space time as follows:

$$ds^2 = dz^2 + \mathrm{e}^{2A(z,\sigma)}\sum_{i,j=1}^{4}\tilde{g}_{ij}dx^i dx^j\,, \qquad \tilde{g}_{\mu\nu}dx^\mu dx^\nu \equiv l^2\left(d\sigma^2 + d\Omega_3^2\right). \tag{2.34}$$

Here the coordinate z is related the coordinate ρ in (2.9) by

$$\rho = \mathrm{e}^{-2z/l}\,. \tag{2.35}$$

Under the choice of metric in (2.34), the curvatures have the following forms:

$$\begin{aligned}
R_{zizj} &= \mathrm{e}^{2A}\left(-A_{,zz} - (A_{,z})^2\right)\tilde{g}_{ij}\,, \\
R_{zA z\sigma} &= -l^2\mathrm{e}^{2A}A_{,z\sigma}g^s_{AB}\,, \\
R_{\sigma A\sigma B} &= \left(-l^2\mathrm{e}^{2A}A_{,\sigma\sigma} - l^2\mathrm{e}^{4A}(A_{,z})^2\right)g^s_{AB}\,, \\
R_{ABCD} &= \left(l^2\mathrm{e}^{2A} - l^2\mathrm{e}^{2A}(A_{,\sigma})^2 - l^4\mathrm{e}^{4A}(A_{,z})^2\right)\left(g^s_{AC}g^s_{BD} - g^s_{AD}g^s_{BC}\right), \\
R_{zz} &= 4\left(-A_{,zz} - (A_{,z})^2\right), \\
R_{z\sigma} &= -3A_{,z\sigma}\,, \\
R_{\sigma\sigma} &= l^2l^2\mathrm{e}^{2A}\left(-A_{,zz} - 4(A_{,z})^2\right) - 3A_{,\sigma\sigma}\,, \\
R_{AB} &= \left(l^2\mathrm{e}^{2A}\left(-A_{,zz} - 4(A_{,z})^2\right) - A_{,\sigma\sigma} - 2(A_{,\sigma})^2 + 2\right)g^s_{AB}\,, \\
R &= -8A_{,zz} - 20(A_{,z})^2 + l^{-2}\mathrm{e}^{-2A}\left(-6A_{,\sigma\sigma} - 6(A_{,\mathrm{sigma}})^2 + 6\right).
\end{aligned} \tag{2.36}$$

Here $\cdot_{,\mu\nu\cdots} \equiv \frac{\partial}{\partial x^\mu}\frac{\partial}{\partial x^\nu}\cdots(\cdot)$. Other curvatures except those obtained by permutating the indeces of the above curvatures vanish. We also write the metric of S_3 in the following form:

$$d\Omega_3^2 = \sum_{A,B=1}^{3} g^s_{AB}dx^A dx^B\,. \tag{2.37}$$

One gets that n^μ and the covariant derivative of n^μ are

$$n^\mu = \delta^\mu_\rho\,, \qquad D_i n^j = \delta^j_i A_{,z}\ (\text{others} = 0)\,. \tag{2.38}$$

Then the actions S in (2.2) and S_b in (2.22) have the following forms:

$$\begin{aligned}
S = l^4\int d^5x\mathrm{e}^{4A}\sqrt{g^s}\Big[&a\left\{64(A_{,zz})^2 + 320A_{,zz}(A_{,z})^2 + 400(A_{,z})^4 + \right.\\
&+ 36l^{-4}\mathrm{e}^{-4A}(A_{,\sigma\sigma})^2 + 72l^{-4}\mathrm{e}^{-4A}A_{,\sigma\sigma}(A_{,\sigma})^2 + 36l^{-4}\mathrm{e}^{-4A}(A_{,\sigma})^4 + \\
&+ l^{-2}\mathrm{e}^{-2A}\left(96A_{,zz} + 240(A_{,z})^2\right)\left(A_{,\sigma\sigma} + (A_{,\sigma})^2 - 1\right) + \\
&\left.+ l^{-4}\mathrm{e}^{-4A}\left(-72A_{,\sigma\sigma} - 72(A_{,\sigma})^2 + 36\right)\right\} + \\
&+ b\left\{20(A_{,zz})^2 + 64A_{,zz}(A_{,z})^2 + 80(A_{,z})^4 + 18l^{-2}\mathrm{e}^{-2A}(A_{,z\sigma})^2 + \right.\\
&+ 12l^{-4}\mathrm{e}^{-4A}(A_{,\sigma\sigma})^2 + 12l^{-4}\mathrm{e}^{-4A}A_{,\sigma\sigma}(A_{,\sigma})^2 + 12l^{-4}\mathrm{e}^{-4A}(A_{,\sigma})^4 + \\
&+ l^{-2}\mathrm{e}^{-2A}\left(12A_{,zz} + 48(A_{,z})^2\right)\left(A_{,\sigma\sigma} + (A_{,\sigma})^2 - 1\right) + \\
&\left.+ l^{-4}\mathrm{e}^{-4A}\left(-12A_{,\sigma\sigma} - 24(A_{,\sigma})^2 + 12\right)\right\} + \\
&+ c\left\{16(A_{,zz})^2 + 32A_{,zz}(A_{,z})^2 + 40(A_{,z})^4 + 24l^{-2}\mathrm{e}^{-2A}(A_{,z\sigma})^2 + \right.\\
&+ 12l^{-4}\mathrm{e}^{-4A}(A_{,\sigma\sigma})^2 + 12l^{-4}\mathrm{e}^{-4A}(A_{,\sigma})^4 + \\
&\left.+ 24l^{-2}\mathrm{e}^{-2A}\ (A_{,z})^2\left(A_{,\sigma\sigma} + (A_{,\sigma})^2 - 1\right) + 12l^{-4}\mathrm{e}^{-4A}\left((-2A_{,\sigma})^2 + 1\right)\right\} + \\
&+ \frac{1}{\kappa^2}\left\{\left(-8A_{,zz} - 20(A_{,z})^2\right) + \right.\\
&\left.+ \left(-6A_{,\sigma\sigma} - 6(A_{,\sigma})^2 + 6\right)\mathrm{e}^{-2A}\right\} + \Lambda\Big]\,,
\end{aligned} \tag{2.39}$$

$$
\begin{aligned}
S_b = l^4 \int d^4x \mathrm{e}^{4A}\sqrt{g^s}\,&\left[16\tilde{a}\left\{\left(-8A_{,zz} - 20\left(A_{,z}\right)^2\right) + \right.\right.\\
&+ \left.\left(-6A_{,\sigma\sigma} - 6\left(A_{,\sigma}\right)^2 + 6\right)\mathrm{e}^{-2A}\right\} A_{,z} + \\
&+ 2\tilde{b}\left\{\left(-20A_{,zz} - 32\left(A_{,z}\right)^2\right) + \left(-6A_{,\sigma\sigma} - 6\left(A_{,\sigma}\right)^2 + 6\right)\mathrm{e}^{-2A}\right\} A_{,z} + \\
&\left. + 32\tilde{c}\left(-A_{,zz} - \left(A_{,z}\right)^2\right) + \eta\right]. \qquad (2.40)
\end{aligned}
$$

¿From the variation over A, one obtains the following equation on the brane, which lies at $z = z_0$:

$$
\begin{aligned}
&\delta\left(S + 2S_b\right) = \\
&= 2V_3 l^4 \int d\sigma \mathrm{e}^{4A}\left[\left(-32\tilde{T} + 24\tilde{U}\right) A_{,z}\delta A_{,zz} + \right.\\
&\quad + \left\{\left(32T - 24U - 32\tilde{T} + 24\tilde{U}\right) A_{,zz} + \left\{\left(32T - 24U - 96\tilde{T} + 72\tilde{U}\right)\left(A_{,z}\right)^2 + \right.\right.\\
&\quad \left.\left. + 12(U - \tilde{U}) l^{-2}\mathrm{e}^{-2A}\left(A_{,\sigma\sigma} + \left(A_{,\sigma}\right)^2 - 1\right) - \frac{8}{\kappa^2} + \frac{8}{\tilde{\kappa}^2}\right\}\delta A_{,z} + \right.\\
&\quad + \left\{\left(64T - 128\tilde{T} + 96\tilde{U}\right) A_{,zz}A_{,z} + \right.\\
&\quad + \left(160T - 128\tilde{T}\right)\left(A_{,z}\right)^3 \times \\
&\quad \times \left(48T - 24\tilde{U}\right) l^{-2}\mathrm{e}^{-2A}\left(A_{,\sigma\sigma} + \left(A_{,\sigma}\right)^2 - 1\right) A_{,z} + \\
&\quad + \left(-36b - 96c - 12\tilde{U}\right) l^{-2}\mathrm{e}^{-2A} A_{,z\sigma\sigma} + \left(-72b - 192c\right) l^{-2}\mathrm{e}^{-2A} A_{,\sigma}A_{,z\sigma} + \\
&\quad \left.\left. + \left(-\frac{8}{\kappa^2} + \frac{32}{\tilde{\kappa}^2}\right) A_{,z} - 4\eta\right\}\delta A\right]. \qquad (2.41)
\end{aligned}
$$

The factors 2 in front of S_b and V_3 come from the fact that we are considering two bulk space (corresponding to $B_5^{(1,2)}$ in [12]) which have one common boundary (S_4 in [12]) or brane.

Here T and $\tilde{T}$ are defined in (2.24) and

$$U = 8a + b\,, \qquad \tilde{U} = 8\tilde{a} + \tilde{b} \qquad (2.42)$$

and V_3 is the volume of the unit 3 sphere:

$$V_3 = \int d^3x_A \sqrt{g^s} = 2\pi^2\,. \qquad (2.43)$$

In order that the variational principle is well defined, the coefficients of $\delta A_{,zz}$ and $\delta A_{,z}$ should vanish. For general A, only one solution is given by the Weyl gravity in (2.5)

$$
\begin{aligned}
&a = \frac{1}{6}\hat{c}\,, \qquad b = -\frac{4}{3}\hat{c}\,, \qquad c = \hat{c}\,, \\
&\tilde{a} = \frac{1}{6}\tilde{\hat{c}}\,, \qquad \tilde{b} = -\frac{4}{3}\tilde{\hat{c}}\,, \qquad \tilde{c} = \tilde{\hat{c}}\,, \\
&\tilde{\kappa}^2 = \kappa^2\,. \qquad (2.44)
\end{aligned}
$$

This remarkable property of Weyl gravity indicates to some natural connection between such version of HD gravity and brane physics.

When A is given by the AdS$_5$ and the brane is S$_4$,

$$A = \ln \sinh \frac{z}{l} - \ln \cosh \sigma \,. \tag{2.45}$$

Equation (2.41) has the following form:

$$\begin{aligned}
&\delta\left(S + 2S_b\right) = \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (2.46)\\
&= 2V_3 l^4 \int d\sigma \frac{\sinh^4 z_0/l}{\cosh^4 \sigma} \left[\left(-32\tilde{T} + 24\tilde{U}\right) \frac{\coth z_0/l}{l} \delta A_{,zz} + \right.\\
&\qquad + \left\{ -\frac{64\tilde{T}}{l^2}\coth^2\frac{z_0}{l} + \frac{32(T-\tilde{T})}{l^2} + \frac{8}{\tilde{\kappa}^2} - \frac{8}{\kappa^2} \right\} \delta A_{,z} + \\
&\qquad \left. + \left\{ -\frac{48\tilde{U}}{l^2 \sinh^2 \frac{z_0}{l}} \left(-\frac{8}{\kappa^2} + \frac{32}{\tilde{\kappa}^2}\right) \frac{\coth z_0/l}{l} - 4\eta \right\} \delta A \right] .
\end{aligned}$$

Then in order that the variational principle is well defined, we obtain

$$\begin{aligned}
0 &= -32\tilde{T} + 24\tilde{U} \,, \\
0 &= \tilde{T} \,, \\
0 &= \frac{32(T-\tilde{T})}{l^2} + \frac{8}{\tilde{\kappa}^2} - \frac{8}{\kappa^2} \,,
\end{aligned} \tag{2.47}$$

or

$$0 = \tilde{T} = \tilde{U} \,, \qquad \frac{1}{\tilde{\kappa}^2} = \frac{1}{\kappa^2} - \frac{4T}{l^2} \,. \tag{2.48}$$

The above results are consistent with those in [20]. Then since η in (2.23) is given by

$$\eta = \frac{6}{l\kappa^2} - \frac{24T}{l^3} \tag{2.49}$$

the equation of motion (in terms of the coefficients) has the following form:

$$0 = \left(\frac{24}{\kappa^2} + \frac{32T}{l^2}\right) \frac{\coth z_0/l}{l} - \frac{24}{l\kappa^2} + \frac{96T}{l^3} \,. \tag{2.50}$$

For the pure Einstein case ($a = b = c = 0$ or $T = 0$), the eq. (2.50) reproduces the previous equation in [12, 22] by putting $\kappa^2 = 16\pi G$. In the pure Einstein case, there is no solution of eq.(2.50). Then for the case, we need to add the quantum correction coming from the trace anomaly of the matter fields on the brane in order that the equation corresponding to (2.50) has a non-trivial solution.

In case of the higher derivative gravity in (2.50), there can be a solution in general. The r.h.s. in eq. (2.50) goes to positive infinity when $z_0 \to +0$ if $24/\kappa^2 + 32T/l^2 > 0$. On the other hand, the r.h.s. becomes $128T/l^2$ when z_0 goes to positive infinity. Then if $T < 0$, there can be a solution in (2.50) without the quantum correction on the brane. As the r.h.s. is the monotonically increasing function of z_0, there is only one solution if $T < 0$. We should also note that there does not appear corrections from R^2 gravity terms for the Weyl gravity (2.5), where $T = 0$. For SG dual of $\mathcal{N} = 2$ Sp(N) theory, from (2.8), we find

$$T = \frac{6N}{24 \cdot 16\pi^2} \,. \tag{2.51}$$

As $T > 0$, there is no classical solution for spherical brane.

If we rewrite (2.50) as

$$\frac{\coth z_0/l}{l} = \frac{24/l\kappa^2 - 96T/l^3}{24/\kappa^2 + 32T/l^2} \,, \tag{2.52}$$

the r.h.s. is the monotonically increasing function of the absolute value $|T|$ of T if $T < 0$. Since $\coth z_0/l$ is the monotonically decreasing function of z_0, the radius $\mathcal{R}$ of S_4, which is given by

$$\mathcal{R} = l\mathrm{e}^{\tilde{A}(y_0)} = l \sinh \frac{z_0}{l} \,, \tag{2.53}$$

decreases if $|T|$ increases when $T < 0$ and l is fixed. We should note that l can be a function of T since l is given by (2.14), which is given in terms of T as follows:

$$l^2 = -\frac{12/\kappa^2 \pm \sqrt{144/\kappa^4 - 32T\Lambda}}{2\Lambda} \,. \tag{2.54}$$

If we fix Λ instead of l, the situation becomes very complicated.

Using $\mathcal{R}$ in (2.53), we can rewrite eq. (2.52) in the following form:

$$0 = \left(\frac{24}{l\kappa^2} + \frac{32T}{l^3}\right)\sqrt{1 + \frac{l^2}{\mathcal{R}^2}} - \frac{24}{l\kappa^2} + \frac{96T}{l^3} \,. \tag{2.55}$$

For SG dual of $\mathcal{N} = 2$ Sp(N) theory, using (2.15) and (2.51), one gets

$$0 = \sqrt{1 + \frac{1}{\mathcal{R}^2}} - 1 + \frac{1}{48N}\sqrt{1 + \frac{1}{\mathcal{R}^2}}\left(5 - \frac{1}{\mathcal{R}^2 + 1}\right) + \frac{22}{48N} + \mathcal{O}\left(N^{-2}\right) . \tag{2.56}$$

In this case, there is no any solution for $\mathcal{R}$. It is remarkable that warped compactification to spherical brane is not realistic in leading (Einstein theory) as well as in next-to-leading order of AdS/CFT correspondence.

Instead of the brane of S_4 in (2.45), we can consider the brane of H_4, where A is given by

$$A = \ln \cosh \frac{z}{l} - \ln \sinh \sigma \,. \tag{2.57}$$

By the similar calculation as for S_4, we again obtain the eqs. (2.48) and (2.49). The equation corresponding to (2.50) has the following form:

$$0 = \left(\frac{24}{\kappa^2} + \frac{32T}{l^2}\right)\frac{\tanh z_0/l}{l} - \frac{24}{l\kappa^2} + \frac{96T}{l^3} \,. \tag{2.58}$$

In case of pure Einstein gravity, there is no solution. When $z_0 = 0$, the r.h.s. in eq. (2.58) becomes $-\frac{24}{l\kappa^2} + \frac{96T}{l^3}$, which can be regarded as negative. On the other hand, when z_0 goes to positive infinity, the r.h.s. becomes $128T/l^2$. Then if $T > 0$, which is different from the case of the S_4 brane, there can be a solution in (2.58) without the quantum correction on the brane. Rewriting eq.(2.58) in the form

$$\frac{\tanh z_0/l}{l} = \frac{24/l\kappa^2 - 96T/l^3}{24/\kappa^2 + 32T/l^2}, \tag{2.59}$$

we find the radius $\mathcal{R}_{\mathrm{H}}$ of H_4, which is defined by

$$\mathcal{R}_{\mathrm{H}} = l\mathrm{e}^{\tilde{A}(y_0)} = l\cosh\frac{z_0}{l}. \tag{2.60}$$

The radius $\mathcal{R}_{\mathrm{H}}$ is monotonically decreasing function of $|T|$ again if $T > 0$ and l is fixed since the l.h.s. in (2.59) is the monotonically increasing function of z_0 and the r.h.s. is the monotonically decreasing function of $|T|$ if $T > 0$.

Using $\mathcal{R}_{\mathrm{H}}$ in (2.60), one can present eq. (2.59) in the following form:

$$0 = \left(\frac{24}{l\kappa^2} + \frac{32T}{l^3}\right)\sqrt{1 - \frac{l^2}{\mathcal{R}_{\mathrm{H}}^2}} - \frac{24}{l\kappa^2} + \frac{96T}{l^3}. \tag{2.61}$$

For SG dual of $\mathcal{N} = 2$ Sp(N) theory, using (2.15) and (2.51), we have

$$0 = \sqrt{1 - \frac{1}{\mathcal{R}_{\mathrm{H}}{}^2}} - 1 + \frac{1}{48N}\sqrt{1 - \frac{1}{\mathcal{R}^2}}\left(5 - \frac{1}{\mathcal{R}_{\mathrm{H}}{}^2 - 1}\right) + \frac{22}{48N} + \mathcal{O}\left(N^{-2}\right). \tag{2.62}$$

For large $\mathcal{R}_{\mathrm{H}}$, eq. (2.56) has the following form:

$$0 = -\frac{1}{2\mathcal{R}_{\mathrm{H}}{}^2} + \frac{2}{3N} + \mathcal{O}\left(\mathcal{R}_{\mathrm{H}}{}^{-4}\right) + \mathcal{O}\left(N^{-2}\right) + \mathcal{O}\left(N^{-1}\mathcal{R}_{\mathrm{H}}{}^{-2}\right) \tag{2.63}$$

or

$$\frac{1}{\mathcal{R}_{\mathrm{H}}^2} = \frac{4}{3N} + \mathcal{O}\left(N^{-2}\right). \tag{2.64}$$

Thus, we demonstrated that next-to-leading order of AdS/CFT correspondence may qualitatively change the results on brane-world cosmology in the leading order. Indeed, in the leading order (Einstein theory) the warped compactification as 5d AdS with hyperbolic brane was impossible. On the same time, account of next-to-leading terms (on the example of particular SCFT dual) improves the situation: creation of hyperbolic brane in 5d AdS space becomes possible.

Let us discuss the situation where the higher derivative gravity in five dimensions corresponds to the Gauss-Bonnet combination which is topological invariant in four deimensions. Then a, b and c are given by

$$a = c = \hat{a}, \qquad b = -4\hat{a}. \tag{2.65}$$

One gets

$$T_{\rm GB} = 3\hat{a}\,. \tag{2.66}$$

Then all the discussion given above can be used by replacing T by $3\hat{a}$ (compare with independent calculation in ref. [21]).

The situation is changed for the case that the brane is R_4, where A is given by

$$A = \frac{z}{l} + \sigma\,. \tag{2.67}$$

Since $A_{,zz} = A_{,\sigma\sigma} = 0$, the coefficient of $\delta A_{,z}$ in (2.41) is given by

$$0 = \frac{32T - 24U - 96\tilde{T} + 72\tilde{U}}{l^2} - \frac{8}{\kappa^2} + \frac{8}{\tilde{\kappa}^2}\,. \tag{2.68}$$

Then we obtain equations weaker than (2.48):

$$\tilde{U} = \frac{4}{3}\tilde{T}\,, \qquad \frac{1}{\tilde{\kappa}^2} = \frac{1}{\kappa^2} - \frac{4T - 3U}{l^2}\,, \tag{2.69}$$

and η in (2.23) is given by

$$\eta = \frac{6}{l\kappa^2} - \frac{24T + 6U - 4\tilde{T}}{l^3}\,. \tag{2.70}$$

The brane equation, which is the coefficient of δA in (2.41) has the following form:

$$\begin{aligned} 0 &= \left(160T - 128\tilde{T}\right)\frac{1}{l^3} + \left(-\frac{8}{\kappa^2} + \frac{32}{\tilde{\kappa}^2}\right)\frac{1}{l} - 4\eta \\ &= \frac{128T + 120U - 144\tilde{T}}{l^3}\,. \end{aligned} \tag{2.71}$$

Then we have

$$\tilde{T} = \frac{8}{9}T + \frac{5}{6}U\,. \tag{2.72}$$

As one sees it admits the number of solutions for very large range of HD terms coefficients. Actually, chosing the suitable surface term the flat brane solution always exists. Thus, we explicitly showed that brane-world cosmology with spherical or hyperbolic or flat brane is possible for big class of HD gravities. The corresponding restrictions to HD terms coefficients are explicitly obtained. The version of HD gravity corresponding to next-to-leading order of AdS/CFT correspondence for specific SCFT is naturally included as sub-class of such theory.

3. Brane-worlds with account of brane quantum matter

In the present section we will discuss the modification of the above scenario in the situation when quantum matter lives on the brane. Of course, bulk dynamics is not touched by brane quantum effects. It is interesting to remark also that in case of AdS/CFT correspondence the explanation of presence of such quantum brane matter effective action naturally appears via holographic renormalization group [23]. In other words, the two dual descritions (SG dual and QFT one) could be patched together into the unique global description of some RG flow [23]. Of course, we will consider general situation when for general HD gravity with arbitrary coefficients some quantum CFT lives on the brane.

The quantum correction induced by the trace anomaly of the free conformally invariant matter fields on the brane can be realized by adding the following effective action W to $S + S_b$:

$$\begin{aligned} W = \hat{b} \int d^4x \sqrt{\widetilde{g}} \widetilde{F} A + \\ + b' \int d^4x \sqrt{\widetilde{g}} \left\{ A \left[2\widetilde{\Delta}^2 + \widetilde{R}_{\mu\nu} \widetilde{\nabla}_\mu \widetilde{\nabla}_\nu - \frac{4}{3} \widetilde{R} \widetilde{\Delta}^2 + \frac{2}{3} (\widetilde{\nabla}^\mu \widetilde{R}) \widetilde{\nabla}_\mu \right] A + \right. \\ \left. + \left(\widetilde{G} - \frac{2}{3} \widetilde{\Delta} \widetilde{R} \right) A \right\} - \\ - \frac{1}{12} \left\{ b'' + \frac{2}{3}(b + b') \right\} \int d^4x \sqrt{\widetilde{g}} \left[\widetilde{R} - 6\widetilde{\Delta} A - 6 (\widetilde{\nabla}_\mu A)(\widetilde{\nabla}^\mu A) \right]^2 . \end{aligned} \tag{3.1}$$

In (3.1), one chooses the 4-dimensional boundary metric as

$$g_{(4)\mu\nu} = \mathrm{e}^{2A} \tilde{g}_{\mu\nu} \tag{3.2}$$

and we specify the quantities with $\tilde{g}_{\mu\nu}$ by using $\tilde{\ }$. G ($\tilde{G}$) and F ($\tilde{F}$) are the Gauss-Bonnet invariant and the square of the Weyl tensor:

$$\begin{aligned} G &= R^2 - 4R_{ij}R^{ij} + R_{ijkl}R^{ijkl} , \\ F &= \frac{1}{3}R^2 - 2R_{ij}R^{ij} + R_{ijkl}R^{ijkl} . \end{aligned} \tag{3.3}$$

In the effective action (3.1), with N scalar, $N_{1/2}$ spinor, N_1 vector fields, N_2 (= 0 or 1) gravitons and N_{HD} higher-derivative conformal scalars, $\hat{b}$, b' and b'' are

$$\begin{aligned} \hat{b} &= \frac{N + 6N_{1/2} + 12N_1 + 611N_2 - 8N_{\mathrm{HD}}}{120(4\pi)^2} , \\ b' &= -\frac{N + 11N_{1/2} + 62N_1 + 1411N_2 - 28N_{\mathrm{HD}}}{360(4\pi)^2} , \\ b'' &= 0 . \end{aligned} \tag{3.4}$$

As usually, b'' may be changed by the finite renormalization of local counterterm in gravitational effective action. As we shall see later, the term proportional to $\left\{ b'' + \frac{2}{3}(\hat{b} + b') \right\}$ in (3.1), and therefore b'', does not contribute to the equations of motion. For $\mathcal{N} = 4$ SU(N) SYM theory

$$\hat{b} = -b' = \frac{N^2 - 1}{4(4\pi)^2} , \tag{3.5}$$

and for $\mathcal{N}=2$ Sp(N) theory

$$\hat{b}=\frac{12N^2+18N-2}{24(4\pi)^2}\,,\qquad b'=-\frac{12N^2+12N-1}{24(4\pi)^2}\,. \tag{3.6}$$

Notice that due to the structure of conformal anomaly the next-to-leading term for $\mathcal{N}=4$ super Yang-Mills theory dual is zero. Non-trivial term proportional to third power on curvatures appears in gravitational action as next-to-next-to-leading term. We should also note that W in (3.1) is defined up to conformally invariant functional, which cannot be determined from only the conformal anomaly. The conformally flat space is an example where anomaly induced effective action is defined uniquely. However, one can argue that such conformally invariant functional is irrrelevant for us because it does not contribute to brane dynamics (does not depend on A).

In the choice of the metric (2.34), we find $\tilde{F}=\tilde{G}=0$, $\tilde{R}=6/l^2$ etc. and (3.1) looks

$$W=V_3\int d\sigma\left[b'A\left(2A_{,\sigma\sigma\sigma\sigma}-8A_{,\sigma\sigma}\right)-2(b+b')\left(1-A_{,\sigma\sigma}-(A_{,\sigma})^2\right)^2\right]. \tag{3.7}$$

Under the variation over A, the change of W is given by

$$\begin{aligned}\delta W &= V_3 l^4\int d\sigma\Big\{4b'\left(A_{,\sigma\sigma\sigma\sigma}-4A_{,\sigma\sigma}\right)- \\ &\qquad -4(\hat{b}+b')\left(A_{,\sigma\sigma\sigma\sigma}+2A_{,\sigma\sigma}-6(A_{,\sigma})^2A_{,\sigma\sigma}\right)\Big\}\delta A\,. \end{aligned} \tag{3.8}$$

Then by substituting the solution (2.45), we find eq. (2.50) is changed as

$$0=2\left\{\left(\frac{24}{\kappa^2}+\frac{32T}{l^2}\right)\frac{\coth z_0/l}{l}-\frac{24}{l\kappa^2}+\frac{96T}{l^3}\right\}\sinh^4\frac{z_0}{l}+24b'\,. \tag{3.9}$$

Using the radius $\mathcal{R}$ of S_4, which is given in (2.53), eq. (3.9) is rewritten

$$0=2\left\{\left(\frac{24}{l\kappa^2}+\frac{32T}{l^3}\right)\sqrt{1+\frac{l^2}{\mathcal{R}^2}}-\frac{24}{l\kappa^2}+\frac{96T}{l^3}\right\}\mathcal{R}^4+24b'\,. \tag{3.10}$$

For H_4 brane, using the radius $\mathcal{R}_{\mathrm{H}}$ in (2.60), one gets

$$0=2\left\{\left(\frac{24}{l\kappa^2}+\frac{32T}{l^3}\right)\sqrt{1-\frac{l^2}{\mathcal{R}_{\mathrm{H}}^2}}-\frac{24}{l\kappa^2}+\frac{96T}{l^3}\right\}\mathcal{R}_{\mathrm{H}}^4+2b'\,. \tag{3.11}$$

For R_4 brane, the eq. (2.71) or (2.72) is not changed.

For the case of S_4, the l.h.s. of (3.10) goes to $24b'$ when $\mathcal{R}\to 0$ and behaves as $\frac{256T}{l^3}\mathcal{R}^4$ for large $\mathcal{R}$ if $T\neq 0$. Furthermore if $T>0$, the l.h.s. of (3.10) is the monotonically increasing function of $\mathcal{R}$. Then if $T>0$, $b'<0$, there is a unique solution. Since when $T > 0$ there is no solution for the classical case in (2.50), the solution for $T > 0$ is generated by the quantum brane matter effects. We should note that even if $b' > 0$ there is always a solution again if $T < 0$. Here, HD gravity plays the essential role. On the other hand even if $b' < 0$ and $T < 0$, the l.h.s. of (3.10) has a unique maximum as a function of $\mathcal{R}$. Then if the value of the maximum is positive, there are two solutions for $\mathcal{R}$, which satisfies (3.10). Since the l.h.s. of (3.10) is the monotonically increasing function of $\mathcal{R}$ when $T > 0$, there is no any solution if $T > 0$ and $b' > 0$.

For the case of H_4, the situation does not change if compare with S_4 if $T \neq 0$. This is because the behavior of the r.h.s. in (3.11) is again governed by the sign of T when $\mathcal{R}_{\mathrm{H}}$ is large. Then if $b' < 0$ and $T > 0$ or if $b' > 0$ and $T < 0$, there is always a solution. The solution for $T < 0$ is generated by the brane matter quantum effects. If $T < 0$ and $b' < 0$, there can be two (quantum) solutions. If $b' < 0$ and $T < 0$, there is no any solution.

The interesting example is provided by $\mathcal{N} = 2$ Sp(N) theory in the situation when SG dual and QFT descriptions are matched together via holographic RG, in both cases in next-to-leading order of AdS/CFT correspondence. In other words, using (2.15), (2.51) (SG dual up to next-to-leading order) and (3.6) (conformal anomaly for SCFT), in eqs. (3.10) and (3.11) leads to

$$\begin{aligned} 0 =& \left\{\sqrt{1+\frac{1}{\mathcal{R}^2}} - 1 + \frac{1}{48N}\sqrt{1+\frac{1}{\mathcal{R}^2}}\left(5 - \frac{1}{\mathcal{R}^2+1}\right) + \frac{22}{48N}\right\}\mathcal{R}^4 - \\ &- \frac{1}{384} - \frac{1}{384N} + \mathcal{O}\left(N^{-2}\right), \qquad (3.12) \\ 0 =& \left\{\sqrt{1-\frac{1}{{\mathcal{R}_{\mathrm{H}}}^2}} - 1 + \frac{1}{48N}\sqrt{1-\frac{1}{\mathcal{R}^2}}\left(5 - \frac{1}{{\mathcal{R}_{\mathrm{H}}}^2-1}\right) + \frac{22}{48N}\right\}{\mathcal{R}_{\mathrm{H}}}^4 - \\ &- \frac{1}{384} - \frac{1}{384N} + \mathcal{O}\left(N^{-2}\right). \qquad (3.13) \end{aligned}$$

Since $T > 0$ and $b' < 0$, there are always solutions in (3.12) and (3.13) for finite $\mathcal{R}$ and $\mathcal{R}_{\mathrm{H}}$. When the brane is S_4 in (3.12), $\mathcal{R}$ becomes $\mathcal{O}(1)$ and the higher derivative terms give a correction of $\mathcal{O}\left(N^{-1}\right)$. When $N \to \infty$, the solution of (3.12) is numerically given by $\mathcal{R}^2 = 0.020833\ldots$ Substituting this value into the r.h.s. of (3.12), we find that it takes a negative value of $-0.00215076/N$. Since $T > 0$, the r.h.s. is monotonically increasing function of $\mathcal{R}$ and goes negative when $\mathcal{R} \to 0$. Then the above result tells that the correction of $\mathcal{O}\left(N^{-1}\right)$ makes $\mathcal{R}$ large. Since $1/\mathcal{R}$ corresponds to the rate of inflation when we Wick-rotate S_4 to de Sitter space, the correction makes the rate small. This is some indication that realistic inflationary cosmology may not be comfortable with warped compactification in AdS/CFT correspondence.

For the brane of H_4, the quantum correction of $\mathcal{O}(N^{-2})$ to $1/\mathcal{R}_{\rm H}^2$ of the classical solution in (2.64) exists but since further higher derivative gravity terms like R^4 also give the contribution of $\mathcal{O}(N^{-2})$, the correction is beyond the control.

Hence, we demonstrated that role of quantum brane matter may be in the significant change of bulk/boundary structure. As we saw there exists the range of HD terms coefficients for which the creation of inflationary or hyperbolic Universe living in d5 AdS is caused exclusively by brane quantum effects. It could be also relevant in frames of AdS/CFT set-up where correct holographic RG description shows the necessity of anomaly induced effective action of brane CFT. In its own turn, the corresponding quantum effects change the brane structure and indicate (despite the negative results of leading order analysis) to the possibility of quantum creation of inflationary brane in d5 AdS space in the next-to-leading order of AdS/CFT correspondence.

4. Discussion

In summary, we investigated brane-world Universe solutions (of special form) for five-dimensional higher derivative gravity. It is shown that such Universe occurs for range of theory parameters. As brane part may be given by de Sitter space which after analytical continuation to lorentzian signature represents ever expanding inflationary Universe then such configuration could be relevant to observable world. The particular examples of Weyl, Gauss-Bonnet or SG dual to some SCFT are also examinated. The role of brane quantum CFT is investigated in the quantum creation of spherical or hyperbolic brane Universes.

There are few interesting topics which may be left for future studies. First of all, one has to investigate the structure of HD propagator near brane in order to understand in detail how HD gravity is trapped. In other words, graviton profile and corrections to Newton potential should be estimated. Second, the dilaton may be included into the analysis of this paper. However, the number of HD terms in dilatonic gravity grows significally. As a result, the analysis is getting too complicated technically. Nevertheless, it could be done at least for some truncated versions of HD dilatonic gravity (say, conformally invariant theory or dilatonic Gauss-Bonnet). Note also that some versions of such theory may be considered as SG duals for non-commutative (super) Yang-Mills theory (presumbly in next-to-leading order). Third, other cosmologies may be considered in the same fashion where bulk and (or) boundary is modified. In particular, the situation where bulk is AdS black hole and boundary is some FRW Universe (or vice-versa) deserves careful study. Fourth, it would be interesting to discuss the cosmological perturbations around our background and the details of late-time inflation. For example, in Einstein gravity the domain wall CFT significally suppresses the metric perturbations [12]. What will be the role of HD gravitational terms in such phenomenon?

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